

① $y_{n+1} = y_n + h f(x_n)$

$x = 2$

$y_1 = 3$

$h = 0.5$

$y_2 = 3 + 0.5 \times \frac{1}{\sqrt{2+2}}$
 $= 3.25$

$x_2 = 2.5$

$y_2 = 3.25$

$h = 0.5$

$y_3 = 3.25 + 0.5 \times \frac{1}{\sqrt{2+0.5}}$
 $= 3.4857\dots$

② a) $\alpha + \beta = -b/a = -3/2$

$\alpha\beta = c/a = 3/4$

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
 $= (-3/2)^2 - 2(3/4) = 3/4$

c) $\boxed{\text{SUM}}$ $3\alpha - \beta + 3\beta - \alpha = 2\alpha + 2\beta$
 $= 2(\alpha + \beta)$
 $= 2(-3/2) = -3$

$\boxed{\text{PRODUCT}}$ $(3\alpha - \beta)(3\beta - \alpha) = 9\alpha\beta - 3\alpha^2 - 3\beta^2 + \alpha\beta$
 $= 10\alpha\beta - 3(\alpha^2 + \beta^2)$
 $= 10(3/4) - 3(3/4) = 21/4$

$x^2 - \boxed{\text{SUM}}x + \boxed{\text{PRODUCT}} = 0$

$\rightarrow x^2 + 3x + 21/4 = 0$

$\rightarrow 4x^2 + 12x + 21 = 0$

③ a) $(2-i)(2^* - i)$

$= 22^* - 2i - 2^*i + i^2$

$= (x+iy)(x-iy) - (x+iy)i - (x-iy)i - 1$

$= x^2 - xyi + xyi - iy^2 - xi - yi^2 - xi + y^2 - 1$

$= x^2 + y^2 - 2xi - 1$

REAL $x^2 + y^2 = 1$

IMAG $-2xc$

b) **REAL** $x^2 + y^2 = 1 = 24$

IMAG $-2xc = -8 \rightarrow x = 4$

$\therefore 4^2 + y^2 = 24$

$y^2 = 9 \rightarrow y = \pm 3$

$\therefore z = (4 + 3i)$ or $z = (4 - 3i)$

(14) a) $y = ab^x$

$\log_{10}(y) = \log_{10}(ab^x)$

$\log_{10}(y) = \log_{10}(a) + x \log_{10}(b)$

$Y = c + mx$

$c = \log_{10}(a)$

$m = \log_{10}(b)$

$\rightarrow a = 10^c$

$\rightarrow b = 10^m$

b)



$x=1, y=12 \rightarrow Y = 1.07918...$

$x=5, y=27 \rightarrow Y = 1.43136...$

c) i) $x=3, Y=1.25 \rightarrow y = 10^{1.25} = 17.78...$
 $= 18$ (2sf)

ii) $a = 10^c = 10^1 = 10$ (y int = 1)

(5) a) $\theta = 2n\pi \pm \alpha$

Key angle $= \cos^{-1}(\sqrt{3}/2) = \pi/6$

$\rightarrow 3\theta = \pi/6 \pm 2n\pi \pm \pi/6$

$\rightarrow 3\theta = 2n\pi + \pi/6 \pm \pi/6$

$\rightarrow \theta = \frac{2}{3}n\pi + \pi/18 \pm \pi/18$

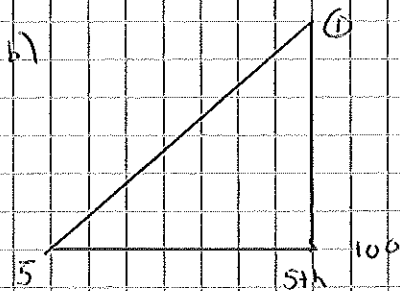
b) Try $n = 7 \rightarrow \omega = 4\frac{3}{4}\pi$ (too small)

Try $n = 8 \rightarrow \omega = 6\frac{2}{3}\pi$, taking -ve

≈ 16.755

$\therefore n = 8$

⑥ a) $5^3 + 3(5^2)h + 3(5)(h^2) + h^3$
 $= 125 + 75h + 15h^2 + h^3$



① $= (5+h)^3 - (5+h)^2$
 $= h^3 + 15h^2 + 75h + 125 - (h^2 + 10h + 25)$
 $= h^3 + 14h^2 + 65h + 100$

Gradient $= \frac{h^3 + 14h^2 + 65h + 100 - 100}{h}$
 $= h^2 + 14h + 65$

ii) As $h \rightarrow 0$, gradient $\rightarrow 65$

This is the gradient of the curve.

⑦ a) i) A^2

$$\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}$$

ii) A^3

$$\begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = 8I$$

b) i) $8I$ = Enlargement, scale factor 8, centre $(0,0)$

ii) $A = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ SF must be $\sqrt[3]{8} = 2$

$\rightarrow 2 \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$

$$= 2 \begin{bmatrix} -\cos(60^\circ) & -\sin(60^\circ) \\ \sin(60^\circ) & -\cos(60^\circ) \end{bmatrix}$$

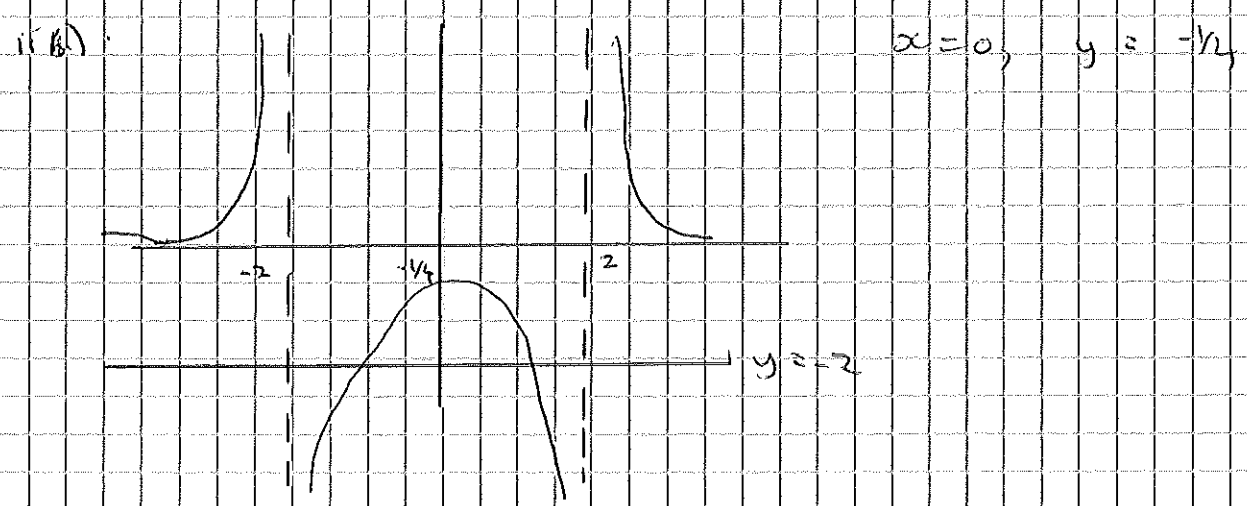
$$= 2 \begin{bmatrix} \cos(120^\circ) & -\sin(120^\circ) \\ \sin(120^\circ) & \cos(120^\circ) \end{bmatrix}$$

= Enlargement, SF 2

Rotation ~~120°~~ anti-clockwise.
120°

8) a) i) Denominator: $x = 2, x = -2$

Numerator: as $x \rightarrow \infty, y \rightarrow 0 \Rightarrow y = 0$



a) Need crossing points:

$$\frac{1}{x^2 - 4} = -2$$

$$\rightarrow 1 = -2x^2 + 8$$

$$2x^2 = 7$$

$$x^2 = 7/2$$

$$x = \pm \sqrt{7/2}$$

curve below line: $-2 < x < -\sqrt{7/2}$

and $\sqrt{7/2} < x < 2$

9) a) i) As it's a reflection in $y = 0x$, it crosses $y = x$

$$\rightarrow xc = \sqrt{8} x^2$$

$$8x = x^2$$

$$x^2 - 8x = 0$$

$$x(x - 8) = 0$$

$$x = 0$$

Origin

$$x = 8 = \text{Point A}$$

$$y = 8 \rightarrow (8, 8)$$

ii) $x \leftrightarrow y$ swap $\rightarrow xc = \sqrt{8} y^2$

iii) Symmetry means solutions lie on perpendicular line

$$\rightarrow \text{gradient of } -1 \quad (y = -x + c)$$

$$b) y = -x + c \rightarrow -x + c = \sqrt{8} x^2$$

$$+8x + 8c = x^2$$

$$x^2 + 8x - 8c = 0$$

For 2 crossing points, $b^2 - 4ac > 0$

$$\rightarrow 8^2 - 4(1)(-8c) > 0$$

$$64 + 32c > 0$$

$$32c > -64$$

$$\rightarrow c > -2 \quad (-64/32)$$

ii) When L touches curve, $b^2 - 4ac = 0 \rightarrow c = -2$

$$\rightarrow y = -x - 2$$

$$\text{Sub into (P)} \rightarrow -x - 2 = \sqrt{8} x^2$$

$$-8x - 8 = x^2$$

$$x^2 + 8x + 8 = 0$$

$$\rightarrow (x + 4)(x + 2) = 0$$

$$\rightarrow x = -4, \quad y = -(-4) - 2 = 2 \rightarrow (-4, 2)$$

Other co-ordinates are reflection in $y = x \rightarrow (4, -2)$